MENIIT

NEET | IIT-JEE | FOUNDATION

Corporate Office: 44-A/1, Kalu Sarai, New Delhi 110016 | Web: www.meniit.com

JEE MAIN-2021

COMPUTER BASED TEST (CBT)

DATE : 31-08-2021 (EVENING SHIFT) | TIME : (3.00 pm to 6.00 pm)

Duration 3 Hours | Max. Marks : 300

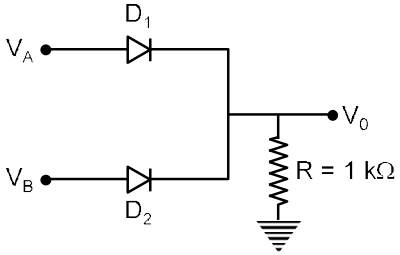
QUESTION & SOLUTIONS

PART A : PHYSICS

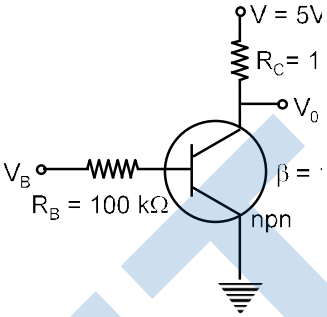
Single Choice Type

This section contains **20 Single choice questions**. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

1. If V_A and V_B are the input voltages (either 5V or 0 V) and V_0 is the output voltage then the two gates represented in the following circuits (A) and (B) are :



(A)



(B)

(1) AND and NOT Gate (2) OR and NOT Gate (3) AND and OR Gate (4) NAND and NOR Gate

Ans. (2)

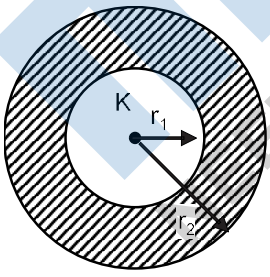
Sol. OR and NOT

2. Two thin metallic spherical shells of radii r_1 and r_2 ($r_1 < r_2$) are placed their centres coinciding. A material of thermal conductivity K is filled in the space between the shells. The inner shell is maintained at temperature θ_1 and the outer shell at temperature θ_2 ($\theta_1 < \theta_2$). The rate at which heat flows radially through the material is :

(1) $\frac{K(\quad_2 \quad_1)(r_2 \quad r_1)}{4 \quad r_1 r_2}$ (2) $\frac{r_1 r_2(\quad_2 \quad_1)}{r_2 \quad r_1}$ (3) $\frac{K(\quad_2 \quad_1)}{r_2 \quad r_1}$ (4) $\frac{4 \quad K r_1 r_2(\quad_2 \quad_1)}{r_2 \quad r_1}$

Ans. (4)

Sol.



Equivalent resistance (R) $\frac{r_2 \quad r_1}{K4 \quad r_2 r_1}$

Thus, Heat flow $\frac{4 \quad K r_1 r_2(\quad_2 \quad_1)}{r_2 \quad r_1}$

3. **Statement-I :** If three forces $\vec{F}_1, \vec{F}_2$ and $\vec{F}_3$ are represented by three sides of triangle and $\vec{F}_1 \quad \vec{F}_2 \quad \vec{F}_3$, then three forces are concurrent forces and satisfy the condition for equilibrium :

Statement-II : A triangle made up of three forces $\vec{F}_1, \vec{F}_2$ and $\vec{F}_3$ as its sides taken in the same order, satisfy the condition for translatory equilibrium. In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Both Statement-I and Statement-II are false.

(2) Both Statement-I and Statement-II are true.

(3) Statement-I is false but Statement-II is true.

(4) Statement-I is true but Statement-II is false.

Ans. (2)

4. A mixture of hydrogen and oxygen has volume 500 cm^3, temperature 300 K, pressure 400 kPa and mass 0.76 g. The ratio of masses of oxygen to hydrogen will be :

(1) 3 : 8 (2) 16 : 3 (3) 3 : 16 (4) 8 : 3

Ans. (2)

Sol. Number of moles of O_2 $\frac{m_1}{32}$

Number of moles of H_2 $\frac{m_2}{2}$

Using ideal gas equation $PV \quad \frac{m_1}{32} \quad \frac{m_2}{2} \quad \frac{25}{3} \quad 300$

$400 \quad 10^3 \quad 500 \quad 10^{-6} \quad \frac{m_1}{32} \quad \frac{m_2}{2} \quad 25 \quad 100 \quad 2 \quad 10^2 \quad \frac{m_1}{m_2} \quad \frac{1}{32} \quad \frac{1}{2} \quad m_2 25 \quad 100$

$\frac{2}{25} \quad \frac{m_1}{m_2} \quad \frac{1}{32} \quad \frac{1}{2} \quad m_2 \qquad$ .....(1)

$m_1 + m_2 = 0.76$

$m_2 \quad 1 \quad \frac{m_1}{m_2} \quad \frac{76}{100} \qquad$ .....(2)

(1) divided by (2) ; $\frac{1 \quad \frac{m_1}{m_2} \quad \frac{1}{32} \quad \frac{1}{2}}{\frac{m_1}{m_2} \quad 1} \quad \frac{2}{25} \quad \frac{100}{76}$

Let assume $\frac{m_1}{m_2} \quad x$; $\frac{x \quad 16}{32[x \quad 1]} \quad \frac{8}{76}$; $76x + 76 \times 16 = 8 \times 32x + 8 \times 32$; $76x + 1216 = 256x + 256$

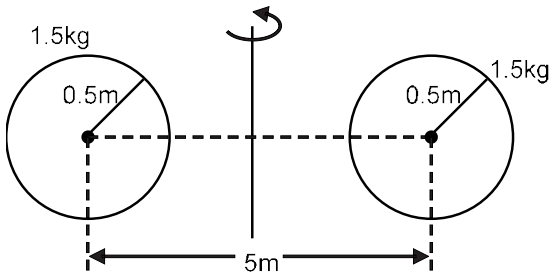
$960 \quad 180x \qquad x \quad \frac{96}{18} \quad \frac{16}{3}$

5. A system consists of two identical spheres each of mass 1.5 kg and radius 50 cm at the ends of a light rod. The distance between the centres of the two spheres is 5 m. What will be the moment of inertia of the system about an axis perpendicular to the passing through its midpoint ?

(1) 1.905×10^5 kgm^2 (2) 19.05 kgm^2 (3) 18.75 kgm^2 (4) 18.75×10^5 kgm^2

Ans. (2)

Sol.



I $\frac{2}{5}MR^2$ Mx^2 2 $\frac{2}{5}$ 1.5 $(0.5)^2$ 1.5 $(2.5)^2$ 2 [0.15 9.375] 2 19.05 kgm^2

6. Consider two separate ideal gases of electrons and protons having same number of particles. The temperature of both the gases are same. The ratio of the uncertainty in determining the position of an electron to that of a proton is proportional to :

(1) $\sqrt{\frac{m_p}{m_e}}$ (2) $\sqrt{\frac{m_e}{m_p}}$ (3) $\frac{m_p}{m_e}$ (4) $\frac{m_p}{m_e}^{3/2}$

Ans. (1)

Sol. K.E. $\frac{3}{2}kT$

$\frac{P^2}{2m}$ $\frac{3}{2}kT$

P $\sqrt{3kTm}$

From Heisenberg uncertainty principle

x P $\frac{h}{4}$

x $\frac{h}{4}$ P $\frac{h}{4\sqrt{3kTm}}$

x $\frac{1}{\sqrt{m}}$

$\frac{x_e}{x_p}$ $\sqrt{\frac{m_p}{m_e}}$

7. Choose the incorrect statement :

(a) The electric lines of force entering into a Gaussian surface provide negative flux.

(b) A charge 'q' is placed at the centre of a cube. The flux through all the faces will be the same.

(c) In a uniform electric field net flux through a closed Gaussian surface containing no net charge, is zero.

(d) When electric field is parallel to a Gaussian surface, it provides a finite non-zero flux.

Choose the most appropriate answer from the options given below :

(1) (b) and (d) only (2) (a) and (c) Only (3) (d) Only (4) (c) and (d) Only

Ans. (3)

8. A bob of mass 'm' suspended by a thread of length ℓ undergoes simple harmonic oscillations with time period T. If the bob is immersed in a liquid that has density $\frac{1}{4}$ times that of the bob and the length of the thread is increased by 1/3rd of the original length, then the time period of the simple harmonic oscillations will be :

(1) T (2) $\frac{4}{3}T$ (3) $\frac{3}{2}T$ (4) $\frac{3}{4}T$

Ans. (2)

Sol. $T \quad 2\sqrt{\frac{\ell}{g}}$

$g_{eff} \quad g \quad \frac{g}{4} \quad \frac{3g}{4}$ $\qquad T' \quad 2\sqrt{\frac{4\ell/3}{3g/4}} \quad 2\sqrt{\frac{16\ell}{9g}}$

$T' \quad \frac{4}{3}T$

9. The magnetic field vector of an electromagnetic wave is given by $B \quad B_0\frac{\hat{i} \quad \hat{j}}{\sqrt{2}}\cos kz \quad t$ where $\hat{i},\hat{j}$ represents unit vector x and y-axis respectively. At t = 0 s, two electric changes q_1 of 4π coulomb and q_2 of 2π coulomb located at $0,0,\frac{}{k}$ and $0,0,\frac{3}{k}$, respectively, have the same velocity of $0.5\ c\hat{i}$, (where c is the velocity of light). The ratio of the force acting on charge q_1 to q_2 is :

(1) $1:\sqrt{2}$ (2) $\sqrt{2}:1$ (3) $2\sqrt{2}:1$ (4) 2 : 1

Ans. (4)

Sol. At t = 0

B at $0,0,\frac{}{k} \quad B_0\frac{\hat{i} \quad \hat{j}}{\sqrt{2}}\cos$

B at $0,0,\frac{3}{k} \quad B_0\frac{\hat{i} \quad \hat{j}}{\sqrt{2}}\cos 3$

Force on charged particle q_1

$F_1 \quad q_1\vec{V}_1 \quad \vec{B}_1$

$$4\ (0.5c\ \hat{i}) \quad B_0\ \frac{\hat{i}\ \ \hat{j}}{\sqrt{2}} \quad \frac{4\ B_0c}{2\sqrt{2}}(\ \hat{k})$$

Force on charged particle q_2

$$F_2 \quad q_2\ \vec{V}_2 \quad \vec{B}_2$$

$$2\ (0.5c\ \hat{i}) \quad B_0\ \frac{\hat{i}\ \ \hat{j}}{\sqrt{2}} \quad \frac{2\ B_0c}{2\sqrt{2}}(\ \hat{k}) \quad \frac{F_1}{F_2} \quad 2$$

10. **Statement-I :** To get a steady dc output from the pulsating voltage received from a full wave rectifier we can connect a capacitor across the output parallel to the load R_L.

Statement-II : To get a steady dc output from the pulsating voltage received from a full wave rectifier we can connect an inductor in series with R_L.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both Statement-I and Statement-II are false.

(2) Statement-I is false but Statement-II is true.

(3) Both Statement-I and Statement-II are true.

(4) Statement-I is true but Statement-II is false.

Ans. (3)

11. Four identical hollow cylindrical columns of mild steel support a big structure of mass 50×10^3 kg. The inner and outer radii of each column are 50 cm and 100 cm respectively. Assuming uniform local distribution, calculate the compression strain of each column. (use $Y = 2.0 \times 10^{11}$ Pa, $g = 9.8\ m/s^2$)

(1) 1.87×10^{-3} (2) 60×10^{-8} (3) 7.07×10^{-4} (4) 2.60×10^{-7}

Ans. (4)

Sol. $$\frac{L}{L} \quad \frac{mg}{4AY}$$

$$\frac{50 \quad 10^3 \quad 9.8}{4\ (100^2 \quad 50^2)\ \ 10^{\ 4} \quad 2 \quad 10^{11}} \quad \frac{5 \quad 10^5}{4 \quad 150 \quad 50 \quad 10^{\ 4} \quad 2 \quad 10^{11}}$$

$$\frac{L}{L} \quad \frac{1}{12} \quad 10^{\ 5} \quad 2.6 \quad 10^{\ 7}$$

12. If velocity [V] time [T] and force [F] are chosen as the base quantities, the dimensions of the mass will be :

(1) $[FVT^{-1}]$ (2) $[FT^{-1}\ V^{-1}]$ (3) $[FT^2\ V]$ (4) $[FTV^{-1}]$

Ans. (4)

Sol. $[M]^1 = [V]^a\ [T]^b\ [F]^c$

$[M]^1 = [LT^{-1}]^a\ [T]^b\ [MLT^{-2}]^c$

$[M]^1 = [M]^c\ [L]^{a+c}\ [T]^{-a+b-2c}$

Thus $C = 1$, $a + c = 0$ $\quad$ $b - a - 2c = 0$

$a = -1$ $\quad$ $b = -1 + 2 = 1$

Thus $[M] = [V]^{-1}\ [T]^1\ [F]^1$

13. A current of 1.5 A is flowing through a triangle, of side 9 cm each. The magnetic field at the centroid of the triangle is : (Assume that the current is flowing in the clockwise direction)

(1) $2\sqrt{3}$ 10 ^{7}T , outside the plane of triangle

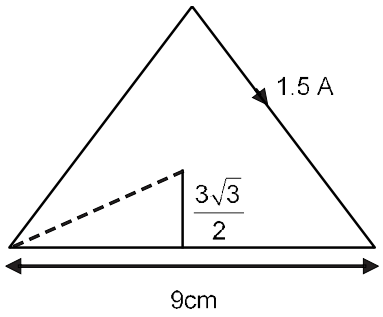
(2) 3×10^{-7} T, outside the plane of triangle

(3) $2\sqrt{3}$ 10 ^{5}T , inside the plane of triangle

(4) 3×10^{-5} T, inside the plane of triangle

Ans. (4)

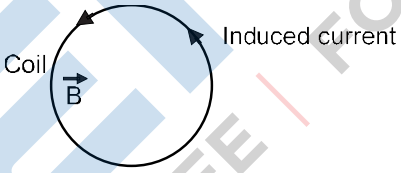
Sol.



$$B_1 \quad \frac{_0 I}{4\ R}(2\sin 60) \quad \frac{4 \quad 10^{\ 7} \quad 1.5}{4 \quad \frac{3\sqrt{3}}{2} \quad 10^{\ 2}} \quad 2 \quad \frac{\sqrt{3}}{2} \quad 1 \quad 10^{\ 5}$$

$B_{net} = 3B_1 = 3 \times 10^{-5}$ T $\otimes$

14. A coil is placed in a magnetic field $\vec{B}$ as shown below :



A current is induced in the coil because $\vec{B}$ is :

(1) parallel to the plane of coil and decreasing with time

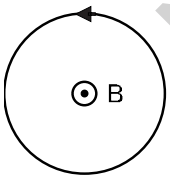
(2) outward and increasing with time

(3) parallel to the plane of coil and increasing with time

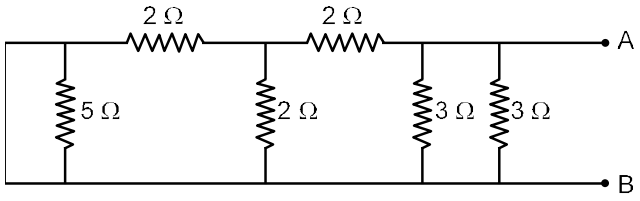
(4) outward and decreasing with time.

Ans. (4)

Sol.



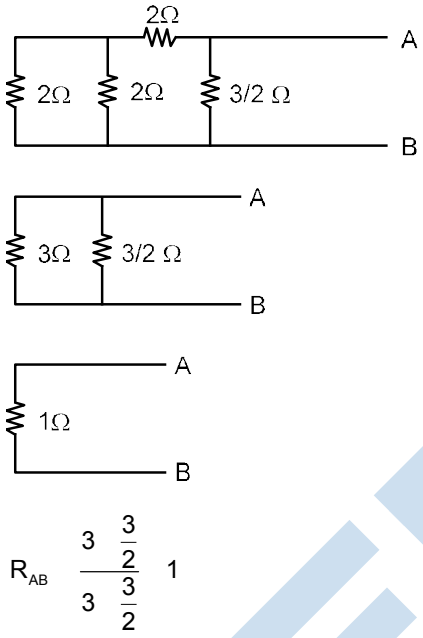
15. The equivalent resistance of the given circuit between the terminals A and B is :



(1) 1 Ω (2) 3 Ω (3) 0 Ω (4) $\frac{9}{2}$

Ans. (1)

Sol.



$$R_{AB} = \frac{3 \cdot \frac{3}{2}}{3 + \frac{3}{2}} + 1$$

16. For a body executing S.H.M. :

(a) potential energy is always equal to its K.E.

(b) Average potential and kinetic energy over any given time interval are always equal.

(c) Sum of the kinetic and potential energy at any point of time is constant.

(d) Average K.E. in one time period is equal to average potential energy in one time period.

Choose the most appropriate option from the options given below :

(1) (c) and (d) (2) only (b) (3) only (c) (4) (b) and (c)

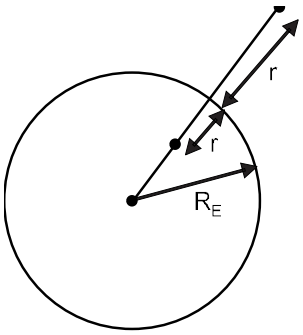
Ans. (1)

17. If R_E be the radius of Earth, then the ratio between the acceleration due to gravity at a depth 'r' below and a height 'r' above the earth surface is : (Given : $r < R_E$)

(1) $1 \quad \frac{r}{R_E} \quad \frac{r^2}{R_E^2} \quad \frac{r^3}{R_E^3}$ (2) $1 \quad \frac{r}{R_E} \quad \frac{r^2}{R_E^2} \quad \frac{r^3}{R_E^3}$ (3) $1 \quad \frac{r}{R_E} \quad \frac{r^2}{R_E^2} \quad \frac{r^3}{R_E^3}$ (4) $1 \quad \frac{r}{R_E} \quad \frac{r^2}{R_E^2} \quad \frac{r^3}{R_E^3}$

Ans. (1)

Sol.



$$\frac{g_h}{g_d} = \frac{\frac{GM}{(R_E + r)^2}}{\frac{GM}{R_E^3}(R_E + r)} = \frac{R_E^2}{(R_E + r)^2(R_E + r)}$$

$$\frac{g_h}{g_d} = \frac{(R_E + r)^2(R_E + r)}{R_E^2} = \frac{R_E^3 + R_E r^2 + r R_E^2 + r^2}{R_E^3} = 1 + \frac{r^2}{R_E^2} + \frac{r}{R_E} + \left(\frac{r}{R_E}\right)^3$$

18. A free electron of 2.6 eV energy collides with a H^+ ion. This results in the formation of a hydrogen atoms in the first excites state and a photon is released. Find the frequency of the emitted photon.

$(h = 6.6 \times 10^{-34}\ J\ s)$

Ans. (2)

Sol. By Energy conservation

$K.E.e = T.E._{H\text{-atom}} + E_{Photon}$

$$2.6 = \frac{13.6}{4} + h\nu$$

$h\nu = 6\ eV$

$$\nu = \frac{6 \times 1.6 \times 10^{-19}}{6.626 \times 10^{-37}}$$

$\nu = 1.45 \times 10^{15}\ Hz = 1.45 \times 10^{9}\ MHz$

19. A block moving horizontally on a smooth surface with a speed of 40 m/s splits into tow parts with masses in the ratio of 1 : 2. If the smaller part moves at 60 m/s in the same direction, then the fractional change in kinetic energy is :

(1) $\frac{1}{8}$ (2) $\frac{2}{3}$ (3) $\frac{1}{3}$ (4) $\frac{1}{4}$

Ans. (1)

Sol. $\boxed{m} \rightarrow 40\ m/s$ $\quad \boxed{2m/3} \rightarrow v$ $\quad \boxed{m/3} \rightarrow 6$

$$m \times 40 = \frac{2}{3}mv + \frac{m}{3} \times 60$$

$120 = 2v + 60$

$v = 30\ m/s$

Now $K_i = \frac{1}{2} m 40^2 = 800\ m$

$K_f = \frac{1}{2}\frac{2m}{3}(30)^2 \frac{1}{2}\frac{m}{3}(60)^2$

$\frac{1}{6}\{m(1800 3600)\} \frac{m(5400)}{6} 900$

$\frac{KE}{K_i} \frac{900m 800m}{800m} \frac{1}{8}$

20. **Statement-I :** Two forces $\vec{P}$ $\vec{Q}$ and $\vec{P}$ $\vec{Q}$ where $\vec{P}$ $\vec{Q}$, when act at an angle θ_1 each other, the magnitude of their resultant is $\sqrt{3\ P^2 Q^2}$, when they act at an angle θ_2, then magnitude of their resultant becomes $\sqrt{2\ P^2 Q^2}$. This is possible only when $\theta_1 < \theta_2$.

Statement-II : In the situation given above.

$\theta_1 = 60^\circ$ and $\theta_2 = 90^\circ$

In the light of the above statement, choose the most appropriate answer from the options given below :

(1) Statement-I is true but Statement-II is false.

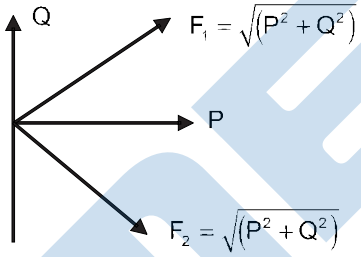
(2) Both Statement-I and Statement-II are true.

(3) Statement-I is false but Statement-II is true.

(4) Both Statement-I and Statement-II are false.

Ans. (2)

Sol.



$\vec{F}_1$ and $\vec{F}_2$ at θ_1

$F_{net1} \sqrt{P^2 Q^2 P^2 Q^2 2(P^2 Q^2)\cos \ _1}$

$F_{net2} \sqrt{P^2 Q^2 P^2 Q^2 2(P^2 Q^2)\cos \ _2}$

If $F_{net1} \sqrt{3(P^2 Q^2)} \sqrt{2(P^2 Q^2) 2(P^2 Q^2)\cos \ _1}$

$\cos \ _1 \frac{P^2 Q^2}{2(P^2 Q^2)}$

$\Rightarrow \theta_1 = 60^\circ$

$F_{net2} \sqrt{2(P^2 Q^2)} \sqrt{2(P^2 Q^2) 2(P^2 Q^2)\cos \ _2}$

Numeric Value Type

This Section contains **10 Numeric Value Type question**, out of 10 only 5 have to be done.

1. In a Young's double slit experiment, the slits are separated by 0.3 mm and the screen is 1.5 m away from the plane of slits. Distance between fourth bright fringes on both sides of central bright fringe is 2.4 cm. The frequency of light used is _____ $\times 10^{14}$ Hz.

Ans. (5)

Sol. The distance between nth bright fridge on both side of central maxima is 2.4 cm. The f = ?

$$y \quad \frac{D}{d} \quad 8$$

$$\frac{d \ y}{8D} \quad \frac{0.3 \quad 10^{3} \quad 2.4 \quad 10^{2}}{8 \quad 1.5} \quad \frac{0.12 \quad 10^{5}}{2} \quad 0.06 \quad 10^{5}$$

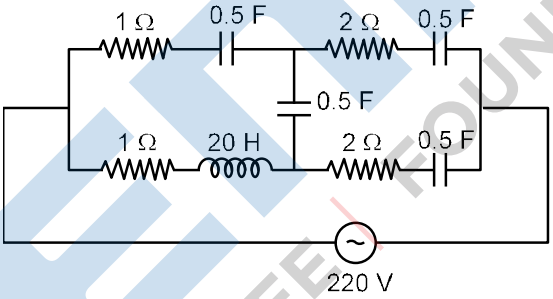
$\lambda = 6 \times 10^{-7}$ m

$$f \quad \frac{C}{6 \quad 10^{7}} \quad 0.5 \quad 10^{15}$$

$f = 5 \times 10^{14}$ Hz

So, x = 5

2. At very high frequencies, the effective impedance of the given circuit will be _____ Ω.

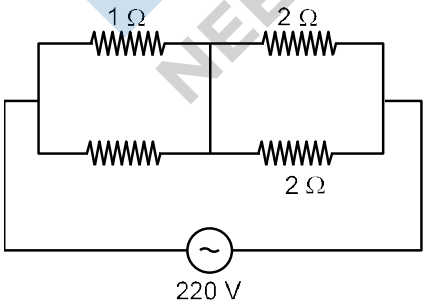


Ans. (2)

Sol. For $\omega \to \infty$

$\frac{1}{C} \to 0 \Rightarrow$ capacitance acts as short circuit

& $\omega L \to \infty$ Inductance acts as open circuit



$$Z \quad 1 \quad \frac{2 \quad 2}{2 \quad 2} \quad 2$$

3. The diameter of a spherical bob is measured using a vernier callipers 9 divisions of the main scale, in the vernier callipers, are equal to 10 divisions of vernier scale. One main scale division is 1 mm. The main scale reading is 10 mm and 8^{th} division of vernier scale was found to coincide exactly with one of the main scale division. If the given vernier callipers has positive zero error of 0.004 cm, then the radius of the bob is _____ $\times 10^{-2}$ cm.

Ans. (52)

Sol. 9MSD = 10 VSD

1 VSD = 0.9 mm

1 MSD = 1mm

$$\text{L.C.} \quad \frac{\text{MSD} \quad \text{VSD}}{\text{MSD}} \quad \frac{1 \quad 0.9}{1} \quad 0.1$$

Reading = Main scale reading + L.C. × vernier scale reading

= 10 + 0.1 × 8

= 10.8

Reading of diameter = 10.8 mm – 0.04 cm

= 10.8 mm – 0.4 mm

= 10.4 mm

radius of the bob = 5.2 mm

= 0.52 cm = 52×10^{-2} cm

4. A sample of gas with $\gamma = 1.5$ is taken through an adiabatic process in which the volume is compressed from 1200 cm^3 to 300 cm^3. If the initial pressure is 200 kPa. The absolute value of the work done by the gas in the process = _____ J.

Ans. (480)

Sol. $V_i = 1200 \times 10^{-6}$ cm^3, $P_i = 200$ KPa

$V_f = 300 \times 10^{-6}$ cm^3, $P_f = ?$

$\gamma = 1.5$

$$P_iV_i \quad P_fV_f$$

$$200 \quad 10^3 \quad \frac{1200 \quad 10^{\ 6}}{300 \quad 10^{\ 6}} \quad P_f$$

$P_f = 200 \times 10^3 \times 4^{3/2}$

$= 200 \times 10^3 \times 8$

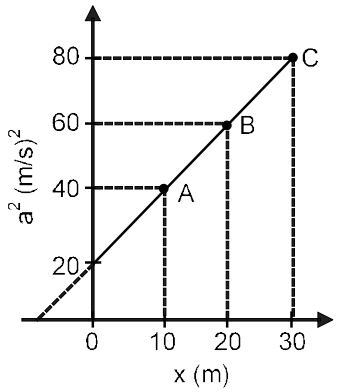
$= 1600 \times 10^3 = 16 \times 10^5$ Pa

$$W \quad \frac{P_iV_i \quad P_fV_f}{1}$$

$$W \quad \frac{2 \quad 10^5 \quad 1200 \quad 10^{\ 6} \quad 16 \quad 10^5 \quad 300 \quad 10^{\ 6}}{1.5 \quad 1} \quad \frac{240 \quad 480}{0.5} \quad 240 \quad 2 \quad 480\,\text{J}$$

|W| = 480J

5. A particle is moving with constant acceleration 'a'. Following graph shows v^2 versus x (displacement) plot. The acceleration of the particle is _____ m/s^2.

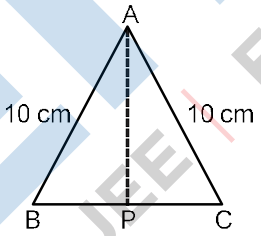


Ans. (1)

Sol. $v^2 = 2x + 20$

$$2V\frac{dv}{dx} \quad 2 \quad a \quad \frac{2}{2} \quad 1\,m/s^2$$

6. Cross-section view of a prism is the equilateral triangle ABC shown in the figure. The minimum deviation is observed using this prism when the angle of incidence is equal to the prism angle. The time taken by light to travel from P (mid point of BC) to A is _____ $\times 10^{-10}$ s. (Given, speed of light in vacuum = 3×10^8 m/s and $\cos 30 \quad \frac{\sqrt{3}}{2}$)



Ans. (5)

Sol. $\delta_{min} = 2i - A$

$A = 2i - A$

$A = i$

$$\frac{\sin\frac{^{min}A}{2}}{\sin\frac{A}{2}} \quad \frac{\sin(A)}{\sin\frac{A}{2}} \quad \frac{\sin 60}{\sin 30}$$

$\sqrt{3}$

$$v \quad \frac{c}{} \quad \frac{3 \quad 10^8}{\sqrt{3}} m/s$$

$$t \quad \frac{AP}{v} \quad \frac{5\sqrt{3} \quad 10^{-2}}{\frac{3 \quad 10^{8}}{\sqrt{3}}} \quad 5 \quad 10^{-10} \text{ sec}$$

So, x = 5

7. A parallel plate capacitor of capacitance 200 μF is connected to a battery of 200 V. A dielectric slab of dielectric constant 2 is now inserted into the space between plates of capacitor while the battery remain connected. The change in the electrostatic energy in the capacitor will be _____ ?

Ans. (4)

Sol. Initially

$C = 200\ \mu F$

$$E_i \quad \frac{1}{2}CV^2 \quad \frac{1}{2} \quad 200 \quad 20^{-6} \quad (200)^2$$

Finally

$C' = KC = 400\ \mu F$

$$E_f \quad \frac{1}{2}CV^2 \quad \frac{1}{2} \quad 400 \quad 10^{-6} \quad (200)^2$$

$$E \quad \frac{1}{2} \quad (400 \quad 200) \quad 10^{-6} \quad 4 \quad 10^{4} \quad 4J$$

8. A long solenoid with 1000 turns/m has a core material with relative permeability 500 and volume 10^3 cm^3. If the core material is replaced by another material having relative permeability of 750 with same volume maintain same current of 0.75 A in the solenoid. The fractional change in the magnetic moment of the core would be approximately $\frac{x}{499}$. Find the value of x.

Ans. (250)

Sol. $\vec{M} \quad \frac{\text{magnetic moment } (\vec{m})}{V}$

$$\vec{H} \quad \frac{\vec{m}}{V}$$

$$\vec{m} \quad (\ _r \quad 1)\, Niv$$

$$\frac{m}{m} \quad \frac{[(750 \quad 1) \quad (500 \quad 1)]\, Niv}{(500 \quad 1)\, Niv} \quad 250$$

9. A resistor dissipated 192 J of energy in 1 s when a current of 4 A is passed through it. Now, when the current is doubled, the amount of thermal energy dissipated in 5 s is _____ J.

Ans. 3840

Sol. $P = i^2Rt$

$192 = 4 \times 4 \times R \times 1$

$R = 12\ \Omega$

$P' = i'^2Rt'$

$= 8 \times 8 \times 12 \times 5 = 3840$ J

10. A bandwidth of 6 MHz is available for A.M. transmission. If the maximum audio signal frequency used for modulating the carrier wave is not to exceed 6 kHz. The number of stations that can be broadcasted within this ban simultaneously without interfering with each other will be _____.

Ans. 500

Sol. Band width = 2 × n × highest modulating frequency.

6MHz = 2 × n × 6 KHz

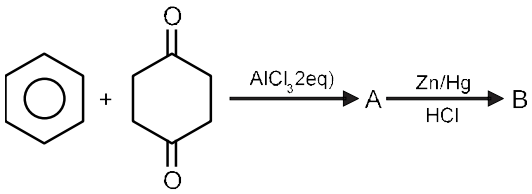
$$n \quad \frac{6 \quad 10^6}{2 \quad 6 \quad 10^3} \quad 500$$

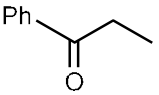
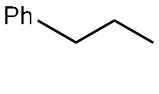
PART B : CHEMISTRY

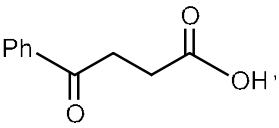
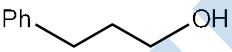
Single Choice Type

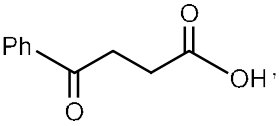
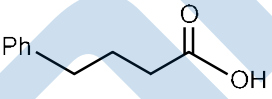
This section contains **20 Single choice questions**. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

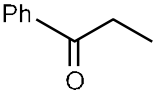
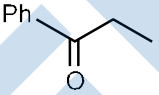
1. The structures of A and B formed in the following reaction are: [Ph=–C_6H_5]



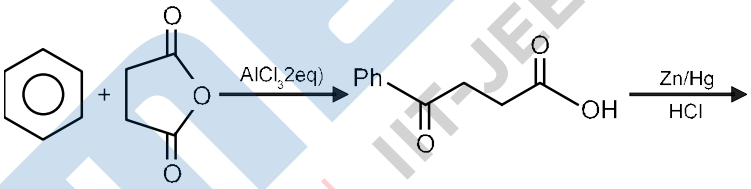
(1) A =  , B = 

(2) A =  , B = 

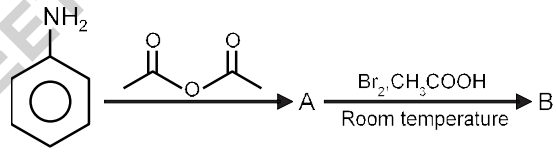
(3) A =  , B = 

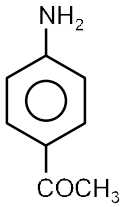
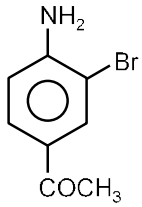
(4) A =  , B = 

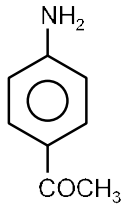
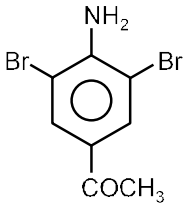
Ans. (3)

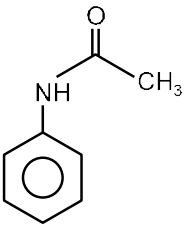
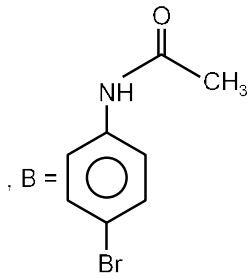
Sol. 

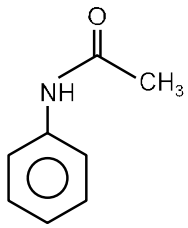
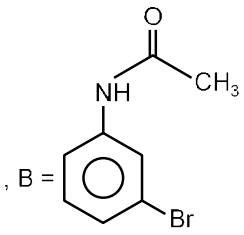
2. The major products A and B formed in the following reaction sequence are:



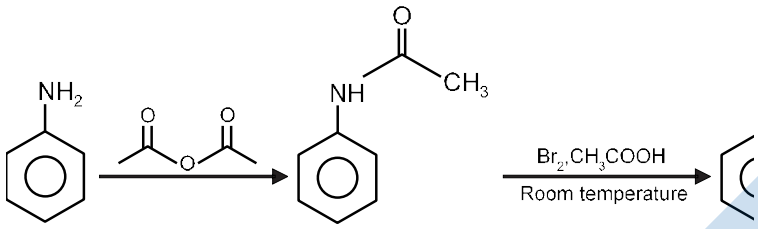
(1) A =  , B = 

(2) A =  , B = 

(3) A =  , B = 

(4) A =  , B = 

Ans. (3)

Sol. 

3. The Eu^{2+} ion is a strong reducing agent in spite of its ground state electronic configuration (outermost) :

[Atomic number of Eu = 63]

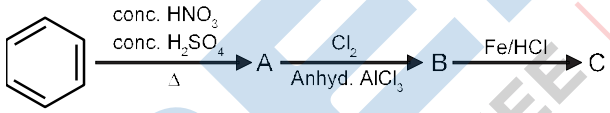
(1) $4f^6 6s^2$ (2) $4f^7$ (3) $4f^7 6s^2$ (4) $4f^6$

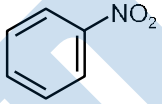
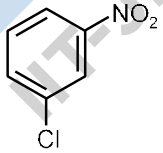
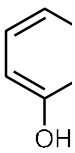
Ans. (2)

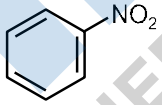
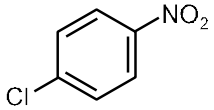
Sol. Eu (Z = 63) = $[_{54}Xe]\ 4f^7\ 6s^2$

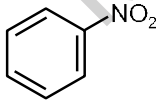
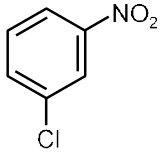
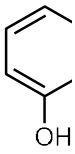
Eu^{+2} (Z = 63) = $[_{54}Xe]\ 4f^7$

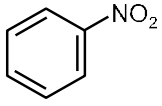
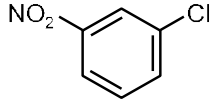
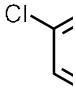
4. Identify correct A, B and C in reaction sequence given below:



(1) A =  B =  C = 

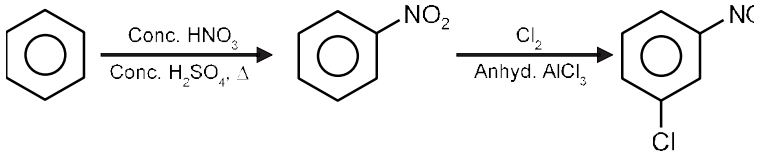
(2) A =  B =  C = 

(3) A =  B =  C = 

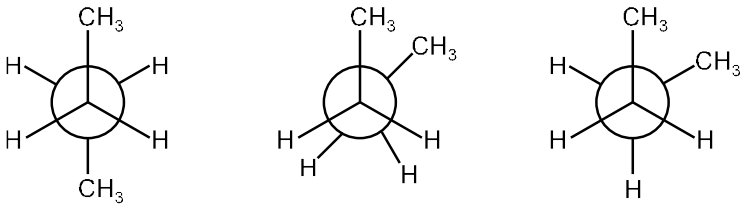
(4) A =  B =  C = 

Ans. (4)

Sol.



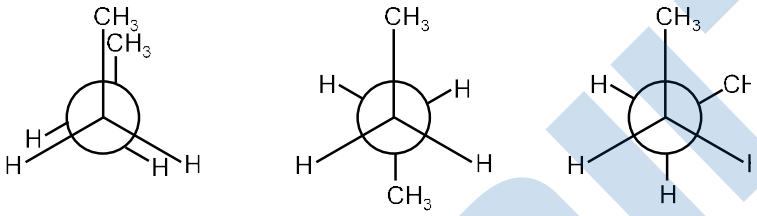
5. Arrange the following conformational isomers of n-butane order of their increasing potential energy:



(1) II < III < IV < I (2) I < III < IV < II (3) I < IV < III < II (4) II < IV < III < I

Ans. (2)

Sol.



Fully eclipsed Anti Gauche

The order Potential energy of above conformations is

Fully eclipsed > Partial eclipsed > Gauche > Anti

6. The incorrect expression among the following is:

(1) $\frac{G_{system}}{S_{Total}}$ (at constant P)

(2) For isothermal process $w_{reversible}$ $nRT\ln\frac{V_f}{V_i}$

(3) $\ln K$ $\frac{H\ \ T\ S}{RT}$

(4) $K = e^{-\Delta G^\circ/RT}$

Ans. (3)

Sol. $\Delta G = \Delta G^\circ + RT \ln Q$ at equilibrium $Q = K_{eq}$ and $\Delta G = 0$

$\Delta G^\circ = -RT \ln K_{eq}$ and $\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$

$\Delta H^\circ - T\Delta S^\circ = -RT \ln K_{eq}$

So, $\ln K_{eq}$ $\frac{H\ \ T\ S}{RT}$ $\frac{H\ \ T\ S}{RT}$

7. Which of the following is NOT an example of fibrous protein?

(1) Collagen (2) Myosin (3) Albumin (4) Keratin

Ans. (3)

Sol. Albumin are fibrous proteins.

8. Which one of the following statements is incorrect?

(1) Bond dissociation enthalpy of H_2 is highest among diatomic gaseous molecules which contain a single bond

(2) Dihydrogen is produced on reacting zinc with HCl as well as NaOH(aq).

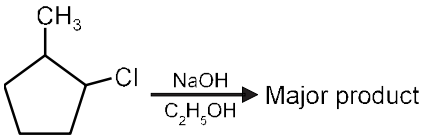
(3) At around 2000 K, the dissociation of dihydrogen into its atoms is nearly 8.1%.

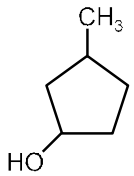
(4) Atomic hydrogen is produced when H_2, molecules at a high temperature are irradiated with UV radiation.

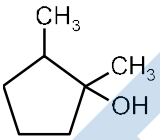
Ans. (3)

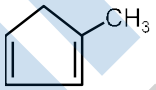
Sol. The dissociation of dihydrogen into its atoms is only ~0.081% around 2000K.

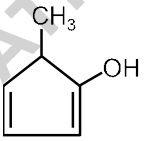
9. The major product of the following reaction is:



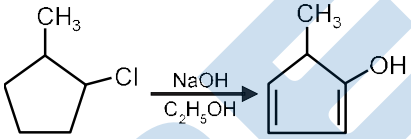
(1) 

(2) 

(3) 

(4) 

Ans. (4)

Sol. 

10. Match List - I with List - II :

	List – I (Parameter)		**List – II (Unit)**
(a)	Cell constant	(i)	S cm^2 mol^{-1}
(b)	Molar conductivity	(ii)	Dimensionless
(c)	Conductivity	(iii)	m^{-1}
(d)	Degree of dissociation of electrolyte	(iv)	Ω^{-1} m^{-1}

Choose the most appropriate answer from the options given below:

(1) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

(2) (a)-(iii), (b)-(i), (c)-(ii), (d)-(iv)

(3) (a)-(i), (b)-(iv), (c)-(iii), (d)-(ii)

(4) (a)-(ii), (b)-(i), (c)-(iii), (d)-(iv)

Ans. (1)

Sol. Cell constant $\frac{\ell}{A}$ m^{1}

Conductivity k $\frac{1}{-}$ $\frac{\ell}{RA}$ $^{1}m^{1}$

Molar Conductivity $_{m}$ $\frac{k \quad 1000}{\text{Molarity}}$ $cm^2\ mole^{1}$

Degree of dissociation = Number of mole dissociated out of one mole

11. Match List - I with List - II :

	List – I (Metal Ion)		**List – II (Group in Qualitative analysis)**
(a)	Mn^{2+}	(i)	Group – III
(b)	As^{3+}	(ii)	Group – IIA
(c)	Cu^{2+}	(iii)	Group – IV
(d)	Al^{3+}	(iv)	Group – IIB

Choose the most appropriate answer from the options given below:

(1) (a)-(i), (b)-(iv), (c)-(ii), (d)-(iii)

(2) (a)-(i), (b)-(ii), (c)-(iii), (d)-(iv)

(3) (a)-(iv), (b)-(ii), (c)-(iii), (d)-(i)

(4) (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i)

Ans. (4)

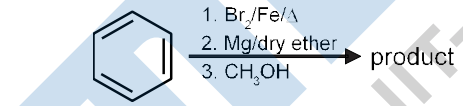
Sol. IIA Group Cations : Hg^{2+}, Pb^{2+}, Bi^{3+}, Cu^{2+}, $*Cd^{2+}$

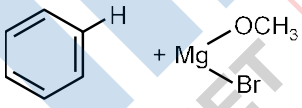
IIB Group Cations : As^{3+}, Sb^{3+}, Sn^{2+}, Sn^{4+}

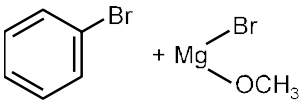
IIIrd Group Cations : Al^{3+}, Cr^{3+}, Fe^{3+}

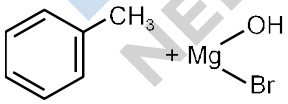
IVth Group Cations : Zn^{2+}, Mn^{2+}, Ni^{2+}, Co^{2+}

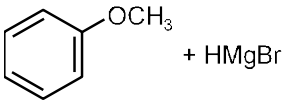
12. For the following sequence of reactions, the correct products are:



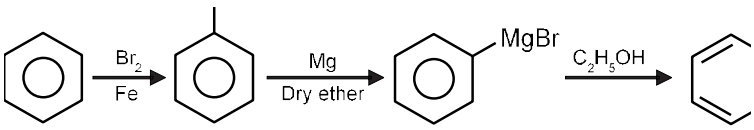
(1) 

(2) 

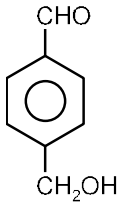
(3) 

(4) 

Ans. (1)

Sol. 

13. For the reaction given below :



The compound which is not formed as a product in the reaction is a:

(1) diol

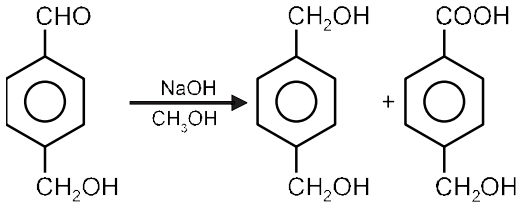
(2) dicarboxylic acid

(3) compound with both alcohol and acid functional groups

(4) monocarboxylic acid

Ans. (2)

Sol.



14. In which one of the following sets all species show disproportionation reaction?

(1) ClO_2, MnO_4 and CrO_7^{2}

(2) MnO_4, ClO_2 and Mn^{3}

(3) ClO_4, MnO_4 and ClO_2

(4) $Cr_2O_7^{2}$, MnO_4, ClO_2 and Cl_2

Ans. (2) (Bonus)

Sol. Disproportionation reactions are a special type of redox reactions. One of the reactants in a disproportionation reaction always contains an element that can exist in at least three oxidation states. The element of reacting species is in intermediate oxidation state and simultaneously gets oxidised and reduced.

Note: NTA answer is (2), but Zigyan ans. is (Bonus)

MnO_4 do not shows disproportionation reaction.

15. Given below are two statement: one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : Lithium salts are hydrated.

Reason (R) : Lithium has higher polarising power than other alkali metal group members. In the light of the above statements, choose the most appropriate answer from the options given below:

(1) (A) is not correct but (R) is correct.

(2) (A) is correct but (R) is not correct.

(3) Both (A) and (R) are correct but (R) is NOT the correct explanation of (A).

(4) Both (A) and (R) are correct and (R) is the correct explanation of (A).

Ans. (3)

Sol. Li salts are hydrated Ex: $LiCl \cdot 6H_2O$

Polarization power $Li^+ > Na^+ > K^+ > Rb^+ > Cs^+$

16. The deposition of X and Y on ground surfaces is referred as wet and dry depositions, respectively. X and Y are:

(1) X = SO_2 ; Y = Ammonium salts (2) X = Ammonium salts ; Y = SO_2

(3) X = Ammonium salts ; Y = CO_2 (4) X = CO_2 ; Y = SO_2

Ans. (2)

Sol. Ammonium salt in rain drop result in wet deposition. Oxides of Nitrogen and sulphur settle down on ground as dry deposition.

17. Which one of the following correctly represents the order of stability oxides X_2O; (X-halogen)?

(1) Br > I > Cl (2) I > Cl > Br (3) Br > Cl > I (4) Cl > I > Br

Ans. (2)

Sol. Iodine oxygen bond is stable due to the greater polarity of bond and stability of chlorine oxygen bond is due to multiple bond formation with orbital of chlorine atom now, Br lacks both characteristics so, the stability order of oxide is given as I > Cl > Br.

18. The number of S =0 bonds present in sulphurous acid peroxodisulphuric acid and pyrosulphuric acid, respectively are

(1) 2, 3 and 4 (2) 2, 4 and 3 (3) 1, 4 and 3 (4) 1, 4 and 4

Ans. (4)

Sol.

Oxiacid of sulphur	Number of S=O Bonds
Pyrosulphuric acid (Oleum) $(H_2S_2O_7)$	4
Peroxodisulphuric acid $(H_2S_2O_6)$	4

 Sulphurous acid (H_2SO_3)	1

19. Which among the following is not a polyester ?

(1) Novolac (2) Dacron (3) PHBV (4) Glyptal

Ans. (1)

Sol. Dacron, PHBV and Glyptal are polyesters.

20. Spin only magnetic moment in BM of $[Fe(CO)_4(C_2O_4)]^+$ is:

(1) 1.73 (2) 0 (3) 1 (4) 5.92

Ans. (1)

Sol. In $[Fe(CO)_4(C_2O_4)]^+$ O. No. of Fe = +3

$_{26}Fe^{+3} = [Ar_{18}]\ 3d^5\ 4s^0$

CO is a S.F.L., so paring is present

$Fe^{+3} = t_{2g}^{2,2,1}\ _{eg}^{0,0}$

number of unpaired electron = 1

so magnetic moment $\sqrt{n(n\ \ 2)}$ B.M.

$\sqrt{1(1\ \ 2)}\ \ \sqrt{3}\ \ 1.73$ B.M.

Numeric Value Type

This Section contains **10 Numeric Value Type question**, out of 10 only 5 have to be done.

1. Sodium oxide reacts with water to produce sodium hydroxide. 200 g of sodium oxide is dissolved in 500 mL of water. Neglecting the change in volume, the concentration of the resulting NaOH solution is_____ $\times 10^{-1}$ M.

[Atomic mass: Na = 23.0, O = 16.0, H = 1.0]

Ans. (13)

Sol. $Na_2O + H_2O \longrightarrow 2NaOH$

According to reaction $\frac{200}{62}$ mole of Na_2O gives $\frac{400}{62}$ mole of NaOH

so molarity of NaOH solution is $\frac{n_{NaOH}}{V_{ml}}$ 1000 $\frac{400}{62}$ $\frac{1000}{500}$ $\frac{800}{62}$ 12.9M 13M

2. The empirical formula for a compound with a cubic close packed arrangement of anions and with cations occupying all the octahedral sites in A_xB. The value of x is_____.

Ans. (1)

Sol. In A_x B

Effective number of B atoms = 4 (in CCP)

Effective number of A atoms = 4 (all O.V.)

So formula of the compound = A_4B_4 = AB

So x = 1

3. The value of magnetic quantum number of the outermost electron of Zn^+ ion is

Ans. (0)

Sol. $Zn^+ = |Ar|_{18}\ 3d^{10}\ 4S^1$

For last e^- : n ℓ m s

4 0 0 $\frac{1}{2}$ Ans. 0

4. In the electrolytic refining of blister copper, the total number of main impurities, from the following, removed as anode mud is

Ph, Sb, Se, Te, Ru, Ag, Au and Pt

Ans. (6)

Sol. Impurities from the blister copper deposit as anode mud which contains antimony, selenium, tellurium, silver, gold and platinum.

5. The transformation occurring in Duma's method is given below.

C_2H_7N 2x $\frac{y}{2}$ Cu xCO$_2$ $\frac{y}{2}H_2O$ $\frac{z}{2}N_2$ 2x $\frac{y}{2}$ Cu

The value of y is_____

Ans. (7)

6. For the reaction A → B, the rate constant k (in s^{-1}) is given by $\log_{10} k = 20.35 - \frac{(2.47 \times 10^3)}{T}$. The energy of activation in kJ mol^{-1} is _____

(Given: R = 8.314 J K^{-1} mol^{-1})

Ans. (47)

Sol. $\log_{10} k = \log A - \frac{E_a}{2.303RT}$

$\log_{10} k = 20.35 - \frac{(2.47 \times 10^3)}{T}$

$\frac{E_a}{2.303R} = 2.47 \times 10^3$

$E_a = \frac{2.47 \times 103 \times 2.303 \times 8.314}{1000} = 47.29 KJ/mole$

7. According to molecular orbital theory, the number of unpaired electron(s) in O_2^2 is

Ans. (0)

Sol. O_2^2 = $18e^-$, there is no unpaired electron

$(\sigma 1s)^2 (\sigma^* 1s)^2 (\sigma 2s)^2 (\sigma^* 2s)^2 (\sigma 2pz)^2 (\pi 2p^2_x = \pi 2p^2_y) (\pi^* 2p_x^2 = \pi^* 2p^2_y) \sigma^* 2p^0_z$

8. CH_4 is adsorbed on 1g charcoal at 0°C following the Freundlich adsorption isotherm 10.0 ml of CH_4 is adsorbed at 100 mm of Hg, whereas 15.0 mL is adsorbed at 200 mm of Hg. The volume of CH_4 adsorbed at 300 mm of Hg is 10x mL. The value of x is_____ x 10^{-2}

[Use $\log_{10}2 = 0.3010$ $\log_{10}3 = 0.4771$]

Ans. (128)

Sol. $\frac{x}{m} = Kp^{\frac{1}{n}}$

$\frac{10}{1} = K(100)^{\frac{1}{n}}$ .....(1)

$\frac{15}{1} = K(200)^{\frac{1}{n}}$ .....(2)

$\frac{V}{1} = K(300)^{\frac{1}{n}}$ .....(3)

Divide (2) to (1) $\frac{15}{10} = 2^{\frac{1}{n}}$

$\log \frac{3}{2} = \frac{1}{n} \log 2$

$\frac{1}{n} = \frac{0.4771 - 0.3010}{0.3010} = 0.585$

Divide (3) to (1) $\frac{V}{10}$ $3^{\frac{1}{n}}$

$\log \frac{V}{10}$ $\frac{1}{n}\log 3$

$\log \frac{V}{10}$ 0.585 0.4771 0.2791

$\frac{V}{10}$ $10^{0.2791}$

$V = 10 \times 10^{0.2791} = 10^{1.2791} = 10^{x}$

$x = 1.2791 = 127.91 \times 10^{-2} \approx 128 \times 10^{-2}$

9. 1.22 g of an organic acid is separately dissolved in 100 g of benzene (K_b = 2.6 K kg mol^{-1}) and 100 g of acetone (K_b = 1.7 K kg mol^{-1}). The acid is known to dimerize in benzene but remain as a monomer in acetone. The boiling point of the solution in acetone increases by 0.17°C. The increase in boiling point of solution in benzene in °C is x × 10^{-2}. The value of x is_____

[Atomic mass: C = 12.0, H = 1.0, 0 = 16.0]

Ans. (13)

Sol. $\Delta T_b = i \times k_b \times$ molality ; m $\frac{w_1}{GMM_1}$ $\frac{1000}{w_2}$

For Acetone solution

0.17 1 1.7 $\frac{1.22}{GMM}$ $\frac{1000}{100}$

GMM of substance = 122 gm/mol

For Benzene solution

T_b i k_b m $\frac{1}{2}$ 2.6 $\frac{1.22}{122}$ $\frac{1000}{100}$ 0.13 13 10^{2} Ans. = 13

10. The pH of solution obtained by mixing 50 mL of 1 M HCl and 30 mL of 1 M NaOH is x × 10^{-4}. The value of x is _____

[log 2.5 = 0.3979]

Ans. (6021)

Sol. Milli equivalents of HCl (N_aV_a) = 50 × 1 = 50

Milli equivalents of NaOH (N_bV_b) = 30 × 1 = 30

Since $N_aV_a > N_bV_b$

[H] $\frac{N_aV_a \quad N_bV_b}{V_a \quad V_b}$ $\frac{50 \quad 30}{80}$ $\frac{20}{80}$ 0.25 2.5 10^{1}

$pH = -\log[H^+] = -\log(2.5 \times 10^{-1}) = 1 - 0.3979 = 0.6021$

$pH \times 10^4 = 0.6021 \times 10^4 = 6021$

PART C : MATHEMATICS

Single Choice Type

This section contains **20 Single choice questions**. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

1. The locus of mid-points of the line segments joining (–3, –5) and the points on the ellipse $\frac{x^2}{4} \quad \frac{y^2}{9} \quad 1$ is

(1) $9x^2 + 4y^2 + 18x + 8y + 145 = 0$ (2) $36x^2 + 16y^2 + 108x + 80y + 145 = 0$

(3) $36x^2 + 16y^2 + 90x + 56y + 145 = 0$ (4) $36x^2 + 16y^2 + 72x + 32y + 145 = 0$

Ans. (2)

Sol. Let points on ellipse $(2\sin\theta, 3\cos\theta)$ and the mid point of line segments joining (–3, –5) and $(2\sin\theta, 3\cos\theta)$ will be (h, k)

then $\frac{2\sin \quad 3}{2} \quad h, \quad \frac{3\cos \quad 5}{2} \quad k$

$2\sin\theta = 2h + 3, \quad 3\cos\theta = 2k + 5$

$\sin \quad \frac{2h \quad 3}{2}, \quad \cos \quad \frac{2k \quad 5}{3} \quad k$

$\therefore \sin^2\theta + \cos^2\theta = 1$

$$\frac{2h \quad 3}{2}^{\ 2} \quad \frac{2k \quad 5}{3}^{\ 2} \quad 1$$

$$\frac{1}{4}[4h^2 \quad 9 \quad 12h] \quad \frac{1}{9}[4k^2 \quad 25 \quad 20k] \quad 1$$

$\Rightarrow 36h^2 + 16k^2 + 108h + 80k + 145 = 0$

So, locus will be

$36x^2 + 16y^2 + 108x + 80y + 145 = 0$

2. Let a_1, a_2, a_3 .....be an A.P. If $\frac{a_1 \quad a_2 \quad \ldots \quad a_{10}}{a_1 \quad a_2 \quad \ldots \quad a_p} \quad \frac{100}{p^2}$, $p \neq 10$, then $\frac{a_{11}}{a_{10}}$ is equal to

(1) $\frac{21}{19}$ (2) $\frac{100}{121}$ (3) $\frac{19}{21}$ (4) $\frac{121}{100}$

Ans. (1)

Sol. $\frac{S_{10}}{S_P} \quad \frac{100}{P^2} \quad S_P \quad \frac{S_{10} \quad P^2}{100}$

$$\frac{a_{11}}{a_{10}} \quad \frac{S_{11} \quad S_{10}}{S_{10} \quad S_9} \quad \frac{S_{10} \quad \frac{121}{100} \quad S_{10}}{S_{10} \quad \frac{S_{10} \quad 81}{100}} \quad \frac{21}{19}$$

3. If $y\frac{dy}{dx} \; x \; \frac{y^2}{x^2} \; \frac{\frac{y^2}{x^2}}{, \; \frac{y^2}{x^2}}$, $x > 0$, $\phi > 0$ and $y(1) = -1$, then $\frac{y^2}{4}$ is equal to

(1) $4\phi(1)$ (2) $4\phi(2)$ (3) $2\phi(1)$ (4) $\phi(1)$

Ans. (1)

Sol. $y\frac{dy}{dx} \; x \; \frac{y^2}{x^2} \; \frac{\frac{y^2}{x^2}}{, \; \frac{y^2}{x^2}}$, Let $\frac{y}{x} \quad t$

$y = xt$

$\frac{dy}{dx} \quad t \quad x \; \frac{dt}{dx}$

$t \; t \quad x\frac{dt}{dx} \qquad t^2 \quad \frac{(t^2)}{'(t^2)}$

$xt\frac{dt}{dx} \quad \frac{(t^2)}{'(t^2)}$

$\frac{t \quad '(t^2)}{(t^2)}dt \quad \frac{1}{x}dx$

Integrating both sides

$\frac{t \quad '(t^2)}{(t^2)}dt \qquad \frac{1}{x}dx$

Let $\phi(t^2) = p$

$\phi'(t^2) .2t = dp$

$\frac{1}{2} \; \frac{1}{p}dp \qquad \frac{1}{x}dx \qquad \frac{1}{2}\ln p \quad \ln x \quad C \quad \frac{1}{2}\ln \; (t^2) \quad \ln x \quad C \quad \frac{1}{2}\ln \quad \frac{y^2}{x^2} \qquad \ln x \quad C$

If $x = 1$, $y = -1$ then $C \quad \frac{1}{2}\ln \quad (1) \quad \frac{1}{2}\ln \quad \frac{y^2}{x^2} \qquad \ln x \quad \frac{1}{2}\ln \quad (1)$

If $x = 3$ then $\ln \quad \frac{y^2}{x^2} \qquad \ln 4 \quad \ln \quad (1)$

So, $\frac{y^2}{x^2} \qquad 4 \; (1)$

4. Negation of the statement $(p \vee r) \Rightarrow (q \vee r)$ is

(1) $p \wedge \sim q \wedge r$ (2) $\sim p \wedge q \wedge r$ (3) $p \wedge q \wedge r$ (4) $\sim p \wedge q \wedge \sim r$

Ans. (1)

Sol. $\equiv \sim ((p \vee r) \rightarrow (q \vee r))$

$\equiv (p \vee r) \wedge (\sim (q \vee r))$

$\equiv (p \vee r) \wedge (\sim q \wedge \sim r)$

$\equiv ((p \vee r) \wedge \sim r) \wedge (\sim q)$

$\equiv ((p \vee \sim r) \vee (r \wedge \sim r)) \wedge (\sim q)$

$\equiv ((p \wedge \sim r) \vee f) \wedge (\sim q)$

$\equiv (p \wedge \sim r) \wedge (\sim q)$

$\equiv p \wedge \sim q \wedge \sim r$

5. The mean and variance of 7 observations are 8 and 16 respectively. If two observations are 6 and 8. then the variance of the remaining 5 observations is

(1) $\frac{134}{5}$ (2) $\frac{112}{5}$ (3) $\frac{536}{25}$ (4) $\frac{92}{5}$

Ans. (3)

Sol. Let a, b, c, d, e be 5 unknown observations.

n = 7, Mean = 8, Variance = 16

$\therefore$ sum of observations = 7 × 8 = 56

$\Rightarrow$ Mean of 5 remaining observations $\frac{56 \quad 8 \quad 6}{5} \quad \frac{42}{5}$

$16 \quad \frac{x_i^2}{7} \quad 64$

$\Rightarrow \Sigma x_i^2 = 560$

$\Rightarrow a^2 + b^2 + c^2 + d^2 + e^2 + 64 + 36 = 560$

$\Rightarrow a^2 + b^2 + c^2 + d^2 + e^2 = 460$

$\therefore$ Variance of remaining 5 observations $\frac{460}{5} \quad \frac{42}{5}^{\,2} \quad \frac{536}{25}$

6. If $\alpha + \beta + \gamma = 2\pi$, then the system of equations

$x + (\cos\gamma)y + (\cos\beta)\, z = 0$

$(\cos\gamma)x + y + (\cos\alpha)\, z = 0$

$(\cos\beta)x + (\cos\alpha)y + z = 0$

has

(1) a unique solution (2) no solution

(3) infinitely many solutions (4) exactly two solutions

Ans. (3)

Sol. $\alpha + \beta + \gamma = 2\pi$ (given)

$$\begin{vmatrix} 1 & \cos & \cos \\ \cos & 1 & \cos \\ \cos & \cos & 1 \end{vmatrix}$$

$= 1 - \cos^2\alpha - \cos^2\beta - \cos^2\gamma + 2\cos\alpha\cos\beta\cos\gamma$

$= \sin^2\alpha - \cos^2\beta - \cos\gamma(\cos\gamma - 2\cos\alpha\cos\beta)$

$= -\cos(\alpha+\beta)\cos(\alpha-\beta) - \cos\gamma(\cos(2\pi - (\alpha+\beta)) - 2\cos\alpha\cos\beta)$

$= -\cos(2\pi-\gamma)\cos(\alpha-\beta) - \cos\gamma\,(\cos(\alpha+\beta) - 2\cos\alpha\cos\beta)$

$= -\cos\gamma\cos(\alpha-\beta) + \cos\gamma\cos(\alpha-\beta)$

$= 0$

So, system of equation has infinitely many solutions

7. Let $\vec{a}, \vec{b}, \vec{c}$ be three vector mutually perpendicular to each other and have same magnitude. If a vector $\vec{r}$ satisfies $\vec{a}\ [(\vec{r}\ \vec{b})\ \vec{a}]\ \vec{b}\ [(\vec{r}\ \vec{c})\ \vec{b}]\ \vec{c}\ [(\vec{r}\ \vec{a})\ \vec{c}]\ 0$ then $\vec{r}$ is equal to

(1) $\frac{1}{2}\ \vec{a}\ \vec{b}\ 2\vec{c}$ (2) $\frac{1}{3}\ \vec{a}\ \vec{b}\ \vec{c}$ (3) $\frac{1}{2}\ \vec{a}\ \vec{b}\ \vec{c}$ (4) $\frac{1}{3}\ 2\vec{a}\ \vec{b}\ \vec{c}$

Ans. (3)

Sol. $(\vec{a}\ \vec{a})(\vec{r}\ \vec{b})\ (\vec{a}\ (\vec{r}\ \vec{b}))\vec{a}\ (\vec{b}\ \vec{b})(\vec{r}\ \vec{c})\ (\vec{b}\ (\vec{r}\ \vec{c}))\vec{b}\ (\vec{c}\ \vec{c})(\vec{r}\ \vec{a})\ (\vec{c}\ (\vec{r}\ \vec{a}))\vec{c}\ 0$

$|\vec{a}|^2(\vec{r}\ \vec{b})\ (\vec{r}\ \vec{a})\vec{a})\ |\vec{b}|^2(\vec{r}\ \vec{c})\ (\vec{r}\ \vec{b})\vec{b})\ |\vec{c}|^2(\vec{r}\ \vec{a})\ (\vec{r}\ \vec{c})\vec{c})\ 0$

$|\vec{a}|^2\ 3\vec{r}\ (\vec{a}\ \vec{b}\ \vec{c})\ (\vec{r}\ \vec{a})\vec{a})\ (\vec{r}\ \vec{b})\vec{b}\ (\vec{r}\ \vec{c})\vec{c})\ 0 \quad |\vec{a}|^2\ |\vec{b}|^2\ |\vec{c}|^2$

$|\vec{a}|^2\ 3\vec{r}\ (\vec{a}\ \vec{b}\ \vec{c})\ (x\vec{a}\ y\vec{b}\ z\vec{c})\ 0$ Let $\vec{r}\ x\vec{a}\ y\vec{b}\ z\vec{c}$

$3\vec{r}\ (\vec{a}\ \vec{b}\ \vec{c})\ \vec{r}\ 0$

$\vec{r}\ \frac{\vec{a}\ \vec{b}\ \vec{c}}{2}$

8. An angle of intersection of the curves $\frac{x^2}{a^2}\ \frac{y^2}{b^2}\ 1$ and $x^2 + y^2 = ab$, $(a > b)$ is

(1) $\tan^{1}\frac{a\ b}{\sqrt{ab}}$ (2) $\tan^{1}\frac{a\ b}{\sqrt{ab}}$ (3) $\tan^{1}\frac{a\ b}{2\sqrt{ab}}$ (4) $\tan^{1}2\sqrt{ab}$

Ans. (2)

Sol. $b^2x^2 + a^2y^2 = a^2b^2$

$(b^2x2 + a^2(ab - x^2)) = a^2b^2$

$x^2\ \frac{ba^2(b\ a)}{b^2\ a^2}\ \frac{a^2b}{a\ b}, y^2\ \frac{ab^2}{a\ b}$

Point of intersection is $\sqrt{\frac{a^2b}{a\ b}}, \sqrt{\frac{ab^2}{a\ b}}$

$\frac{x^2}{a^2}\ \frac{y^2}{b^2}\ 1\ \frac{2x}{a^2}\ \frac{2y}{b^2}\frac{dy}{dx}\ 0$

$m_1 \quad \frac{b^2x}{a^2y}$

$x^2 \quad y^2 \quad ab \quad 2x \quad 2y\frac{dy}{dx} \quad 0 \quad m_2 \quad \frac{x}{y}$

$$\tan \left|\frac{m_1 \quad m_2}{1 \quad m_1m_2}\right| \left|\frac{\frac{b^2x}{a^2y} \quad \frac{x}{y}}{1 \quad \frac{b^2x^2}{a^2y^2}}\right| \left|\frac{xy(a^2 \quad b^2)}{b^2x^2 \quad a^2y^2}\right| \left|\frac{xy(a^2 \quad b^2)}{a^2b^2}\right| \left|\sqrt{\frac{a^3b^3}{(a \quad b)^2}} \quad \frac{(a^2 \quad b^2)}{a^2b^2}\right| \left|\frac{(a \quad b)}{\sqrt{ab}}\right|$$

9. Number of the solutions of the equation $32^{\tan^2 x} \quad 32^{\sec^2 x} \quad 81, 0 \quad x \quad \frac{}{4}$ is

(1) 2 (2) 3 (3) 1 (4) 0

Ans. (3)

Sol. $32^{\tan^2 x} \quad 32^{1 \quad \tan^2 x} \quad 81$

$33 \quad 32^{\tan^2 x} \quad 81$

$32^{\tan^2 x} \quad \frac{27}{11}$

$\tan^2 x \quad \ln_{32} \frac{27}{11}$

$\tan^2 x \quad \sqrt{\ln_{32} \frac{27}{11}} \quad (0,1)$

one solution in $0, \frac{}{4}$

10. The sum of the roots of the equation, $x + 1 - 2\log_2(3 + 2^x) + 2\log_4(10 - 2^{-x}) = 0$ is

(1) $\log_2 14$ (2) $\log_2 12$ (3) $\log_2 13$ (4) $\log_2 11$

Ans. (4)

Sol. $\Rightarrow x + 1 - 2\log_2(3 + 2^x) + 2\log_4(10 - 2^{-x}) = 0$

$x \quad 1 \quad \log_2 \frac{10 \quad 2^x \quad 1}{(3 \quad 2^x)^2} \quad 0$

$1 \quad \log_2 \frac{10 \quad 2^x \quad 1}{(3 \quad 2^x)^2} \quad 0$

$\frac{10 \quad 2^x \quad 1}{9 \quad (2^x) \quad 6 \quad 2^x} \quad \frac{1}{2} \quad (2^x)^2 \quad 14 \quad 2^x \quad 11 \quad 0$

Let $2^x = y$

$\Rightarrow y^2 - 14y + 11 = 0$

$y \quad \frac{14 \quad \sqrt{152}}{2} \quad 7 \quad \frac{\sqrt{152}}{2}$

$y_1 \quad 7 \quad \frac{\sqrt{152}}{2}, y_2 \quad 7 \quad \frac{\sqrt{152}}{2}$

$2^{x_1} \quad 7 \quad \frac{\sqrt{152}}{2}, 2^{x_2} \quad 7 \quad \frac{\sqrt{152}}{2}$

$x_1 \quad \log_2 \ 7 \quad \frac{\sqrt{152}}{2}, x_2 \quad \log_2 \ 7 \quad \frac{\sqrt{152}}{2}$

$x_1 \quad x_2 \quad \log_2 \ 49 \quad \frac{152}{2}$

$[x_1 + x_2 = \log_2 11]$

11. Let A be the set of all points (α, β) such that the area of triangle formed by the points (5, 6), (3, 2) and (α, β) is 12 square units. Then the least possible length of a line segment joining the origin to a point in A, is

(1) $\frac{4}{\sqrt{5}}$ (2) $\frac{12}{\sqrt{5}}$ (3) $\frac{16}{\sqrt{5}}$ (4) $\frac{8}{\sqrt{5}}$

Ans. (4)

Sol. If area is 12 square units then

$$\begin{vmatrix} & & 1 \\ 5 & 6 & 1 \\ 3 & 2 & 1 \end{vmatrix} \quad 24$$

$4\alpha - 2\beta - 8 = \pm 24$

$4\alpha - 2\beta = 32, 4\alpha - 2\beta + 16 = 0$

$2\alpha - \beta - 16 = 0, 2\alpha - \beta + 8 = 0$

Distance from origin

$d \quad \sqrt{^2 \quad 2 \quad 8^2} \quad \sqrt{5^2 \quad 32 \quad 64} \quad (\quad 2 \quad 8)$

$D^2 = 5\alpha^2 + 32\alpha + 64$

$\frac{d \ D^2}{d} \quad 10 \quad 32 \quad 0$

$\frac{16}{5}$

$\frac{32}{5} \quad 8 \quad \frac{8}{5}$

$D \quad \sqrt{\frac{16}{5}^2 \quad \frac{8}{5}^2} \quad \frac{8}{5}\sqrt{5} \quad \frac{8}{\sqrt{5}}$

Similarly if $\beta = 2\alpha - 16$, $D \quad \frac{16}{\sqrt{5}}$,

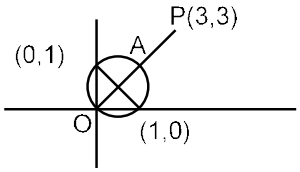
So, least possible length of line segment is $\frac{8}{\sqrt{5}}$

12. If z is a complex number such that $\frac{z\ \ i}{z\ \ 1}$ is purely imaginary, then the minimum value of $|z - (3 + 3i)|$ is

(1) $2\sqrt{2}$ (2) $6\sqrt{2}$ (3) $2\sqrt{2}\ \ 1$ (4) $3\sqrt{2}$

Ans. (1)

Sol. $\frac{z\ \ i}{z\ \ 1}$



Minimum distance is AP = OP – OA

$3\sqrt{2}\ \ \sqrt{2}$

$2\sqrt{2}$

13. Let S = (1,2,3,4,5,6). Then the probability that a randomly chosen onto function g from S to S satisfies g(3) = 2g(1) is

(1) $\frac{1}{15}$ (2) $\frac{1}{10}$ (3) $\frac{1}{5}$ (4) $\frac{1}{30}$

Ans. (2)

Sol. 1 1

2 2

3 3 total onto functions = 6!

4 4

5 5

6 6

9(3) = 2g(1)

2 1

4 2 onto functions for which [g(3)= 2g(1)] = 4!+4!+4!

6 3

$\frac{3\ \ 4!}{30\ \ 4!}\ \ \frac{1}{10}$

14. Let $f : N \to N$ be a function such that $f(m + n) = f(m) + f(n)$ for every $m, n \in N$. If f(6) = 18 then f(2).f(3) is equal to:

(1) 18 (2) 36 (3) 6 (4) 54

Ans. (4)

Sol. $f(3 + 3) = f(3) + f(3) \Rightarrow f(3) = 9$

$f(3) = f(2 + 1) = f(2) + f(1)$

$= f(1+ 1) + f(1)$

$= f(1) + f(1) + f(1)$

$9 = 3f(1) \quad \Rightarrow \quad f(1) = 3$

$f(2) = f(1 + 1) = f(1) + f(1) = 6$

$f(2).\ f(3) = (6)\ (9) = 54$

15. Let f be any continuous function on [0, 2] and twice differentiable on (0, 2). If f(0) = 0, f(1) = 1 and f(2) = 2, then

(1) $f''(x) = 0$ for some $x \in (0, 2)$ (2) $f''(x) = 0$ for all $x \in (0, 2)$

(3) $f'(x) = 0$ for some $x \in [0, 2]$ (4) $f''(x) > 0$ for all $x \in (0, 2)$

Ans. (1)

Sol. f(0) = 0 , f(1) = 1 and f(2) =2

Let h(x) = f(x) –x

clearly h(x) will be continuous and twice different bale on (0,2)

h(0) = h(1) = h(2) = 0

By Rolles mean value theorem in [0, 1]

$h'(C_1) = 0$

$f'(C_1) - 1 = 0 \Rightarrow f'(C_1) = 1$ where $C_1 \in (0,1)$

Also on the interval [1, 2]

$h'(c_2) = 0$

$f'(C_2) - 1 = 0 \Rightarrow f'(C_2) = 1$ where $C_2 \in (1,2)$

Now use Rolls theorem on $[C_1, C_2]$ for $f'(x)$

we have $f''(C) = 0$

$\Rightarrow f''(C) = 0$ where $C \in (C_1, C_2)$

Hence $f''(x) = 0$ for some $x \in (0, 2)$

(A) correct

16. The domain of the function $f(x) \quad \sin^{1} \frac{3x^2 \quad x \quad 1}{x \quad 1^2} \quad \cos^{1} \frac{x \quad 1}{x \quad 1}$ is :

(1) $0, \frac{1}{2}$ (2) $\frac{1}{4}, \frac{1}{2} \quad \{0\}$ (3) $2,0 \quad \frac{1}{4}, \frac{1}{2}$ (4) $0, \frac{1}{4}$

Ans. (2)

Sol. $1 \quad \frac{x \quad 1}{x \quad 1} \quad 1 \qquad 1 \quad 1 \quad \frac{2}{x \quad 1} \quad 1 \quad 2 \quad \frac{2}{x \quad 1} \quad 0 \quad 0 \quad \frac{1}{x \quad 1} \quad 1 \quad x \quad 1 \quad [1, \ )$

$\Rightarrow x \in [0, \infty) \quad \ldots\ldots.(i)$

and $\quad 1 \quad \frac{3x^2 \quad x \quad 1}{(x \quad 1)^2} \quad 1 \quad \Rightarrow -(x-1)^2 \le 3x^2 + x - 1 \le (x-1)^2,\ x \ne 1$

$\Rightarrow -(x^2 - 2x + 1) \le 3x^2 + x - 1$ and $3x^2 + x - 1 \le x^2 - 2x + 1$

$\Rightarrow 4x^2 - x \geq 0$ $\Rightarrow 2x^2 + 3x - 2 \leq 0$

$\Rightarrow x(4x-1) \geq 0$ $\Rightarrow (x+2)(2x-1) \leq 0$

$x \in (-\infty, 0] \cup \left[\frac{1}{4}, \infty\right)$ $x \in [-2, \frac{1}{2}]$

$x \in [-2, 0] \cup \left[\frac{1}{4}, \frac{1}{2}\right]$ ..... (ii)

(i) $\cap$ (ii) we get $x \in \{0\} \cup \left[\frac{1}{4}, \frac{1}{2}\right]$

17. If $\frac{dy}{dx} = \frac{2^x y - 2^y \cdot 2^x}{2^x - 2^{x+y} \log_e 2}$, $y(0) = 0$, then for $y = 1$, the value of x lies in the interval:

(1) (2, 3) (2) (1, 2) (3) $\left(0, \frac{1}{2}\right)$ (4) $\left(\frac{1}{2}, 1\right)$

Ans. (2)

Sol. $\frac{dy}{dx} = \frac{2^x y - 2^y \cdot 2^x}{2^x - 2^{x+y} \log_e 2}$

$\frac{dy}{dx} = \frac{2^x(y - 2^y)}{2^x(1 - 2^y \log_e 2)}$

$\frac{1 - 2^y \log_e 2}{y - 2^y} dy = dx$

$\Rightarrow \ln|y + 2^y| = x + C$

Now $y(0) = 0 \Rightarrow C = 0$

$\ln|y + 2^y| = x$

Now for $y = 1$ we have

$x = \ln(1 + 2) = \ln 3 \in (1,2)$

18. Let $\lim\limits_{x \to \pi/4} \frac{\tan^3 x - \tan x}{\cos\left(x - \frac{\pi}{4}\right)}$ and $\lim\limits_{x \to 0} \cos x^{\cot x}$ are the roots of equation $ax^2 + bx - 4 = 0$, then the ordered pair (a, b) is:

(1) (1, –3) (2) (–1, 3) (3) (1, 3) (4) (–1, –3)

Ans. (3)

Sol. $\lim\limits_{x \to 0} \cos x^{\cot x}$

$e^{\lim\limits_{x \to 0} \frac{\cos x - 1}{\tan x}}$

$e^{\lim\limits_{x \to 0} \frac{\sin x}{\sec^2 x}}$ e^0 1

$$\lim_{x \to \pi/4} \frac{\tan^3 x - \tan x}{\cos\left(x - \frac{\pi}{4}\right)}$$

$$\lim_{x \to \pi/4} \frac{\tan x(\tan x - 1)(\tan x + 1)}{\cos\left(x - \frac{\pi}{4}\right)}$$

$$2\lim_{x \to \pi/4} \frac{\tan x - 1}{\cos\left(x - \frac{\pi}{4}\right)} \quad 2\lim_{x \to \pi/4} \frac{\sec^2 x}{\sin\left(x - \frac{\pi}{4}\right)} \quad \frac{2 \quad 2}{1} \quad 4$$

equation whose roots are α and β is

$x^2 + 3x - 4 = 0$

$\therefore$ a = 1, b = 3

19. If [x] is the greatest integer $\le$ x, then $\int_0^2 2^{\sin\frac{x}{2}} (x - [x])^{[x]}\,dx$ is equal to:

(1) $4(\pi + 1)$ (2) $2(\pi + 1)$ (3) $4(\pi - 1)$ (4) $2(\pi - 1)$

Ans. (3)

Sol. $I = \int_0^2 2^{\sin\frac{x}{2}} (x - [x])^{[x]}\,dx$

$I = \int_0^1 2^{\sin\frac{x}{2}} x^0\,dx + \int_1^2 2^{\sin\frac{x}{2}} (x - 1)^1\,dx$

$I = \int_0^1 2^{\sin\frac{x}{2}}\,dx + \int_1^2 2^{\sin\frac{x}{2}} (x - 1)\,dx$

$I = \left[\frac{2}{-}\cos\frac{x}{2}\right]_0^1 + 2^{(x-1)\frac{2}{-}\cos\frac{x}{2}}\Big|_1^2 + \frac{2}{1}\int_1^2 \cos\frac{x}{2}\,dx$

$I = \frac{2}{-} \quad \frac{2}{-}1 \quad 0 \quad 2 \quad \frac{2}{-}\sin\frac{x}{2}\Big|_1^2$

$I = 2\pi + 2\pi + 4(0 - 1)$

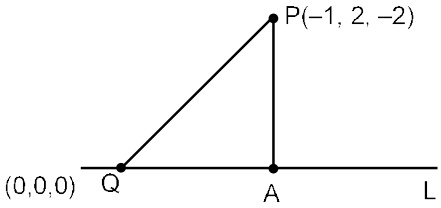
$I = 4\pi - 4$

20. Distance of point (–1, 2, –2) from the line of intersection of the planes 2x + 3y + 2z = 0 and x – 2y + z = 0 is:

(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{\sqrt{42}}{2}$ (3) $\frac{\sqrt{34}}{2}$ (4) $\frac{5}{2}$

Ans. (3)

Sol.



Equation of the intersection line.

$$\frac{x\ 0}{1} \quad \frac{y\ 0}{0} \quad \frac{z\ 0}{1}$$

$AP^2 = PQ^2 - QA^2$

QA = Projection of PQ on line L

$$QA \quad \left|\frac{1\ (0\ 1)\ 0\ (0\ 2)\ 1\ (0\ 2)}{\sqrt{1\ 1}}\right| \quad \frac{1}{\sqrt{2}}$$

$$QA \quad \frac{1}{\sqrt{2}}$$

$$PQ \quad \sqrt{1\ 4\ 4} \quad 3$$

$$AP^2 \quad PQ^2 \quad QA^2 \quad 9 \quad \frac{1}{2}$$

$$AP \quad \frac{\sqrt{17}}{\sqrt{2}} \quad \frac{\sqrt{34}}{2}$$

Numeric Value Type

This Section contains **10 Numeric Value Type question**, out of 10 only 5 have to be done.

1. If $\int \frac{\sin x}{\sin^3 x \cos^3 x}dx = a\log_e|1 - \tan x| - \log_e|1 - \tan x - \tan^2 x| - \tan^{-1}\frac{2\tan x - 1}{\sqrt{3}} + C$, when C is constant of integration, then the value of $18(\alpha + \beta + \gamma^2)$ is

Ans. (3)

Sol. $\int \frac{\sin x}{\sin^3 x \cos^3 x}dx$

$I = \int \frac{\tan x \sec^2 x}{\tan^3 x - 1}dx$

Put $\tan x = t \Rightarrow \sec^2 x\, dx = dt$

$\int \frac{t\,dt}{t^3 - 1} = \int \frac{t}{(t-1)(t^2+t+1)}dt$

Now $\frac{t}{(t-1)(t^2+t+1)} = \frac{A}{t-1} + \frac{Bt+C}{t^2+t+1}$

$\Rightarrow t = A(t^2 - t + 1) + (Bt + C)(t + 1)$

$A = \frac{1}{3}, B = -\frac{1}{3}, C = \frac{1}{3}$

Hence $I = \int \left|\frac{\frac{1}{3}}{t-1} - \frac{\frac{1}{3}t + \frac{1}{3}}{t^2+t+1}\right|dt$

$\frac{1}{3}\ln(t-1) - \frac{1}{3}\int \frac{\frac{1}{2}(2t+1) - \frac{3}{2}}{t^2+t+1}dt$

$\frac{1}{3}\ln(t-1) - \frac{1}{6}\ln(t^2+t+1) + \frac{1}{2}\int \frac{dt}{\left(t+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$

$\frac{1}{3}\ln(t-1) - \frac{1}{6}\ln(t^2+t+1) + \frac{1}{2}\cdot\frac{2}{\sqrt{3}}\tan^{-1}\frac{2t+1}{\sqrt{3}} + C$

$\frac{1}{3}\ln(\tan x - 1) - \frac{1}{6}\ln(\tan^2 x + \tan x + 1) + \frac{1}{\sqrt{3}}\tan^{-1}\frac{2\tan x + 1}{\sqrt{3}} + C$

$\alpha = \frac{1}{3}, \beta = -\frac{1}{6}$ and $\gamma = \frac{1}{\sqrt{3}}$

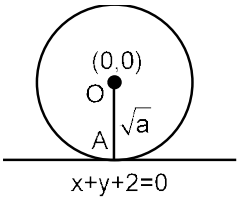
So, $18(\alpha + \beta + \gamma^2) = 18\left(\frac{1}{3} - \frac{1}{6} + \frac{1}{3}\right) = 3$

2. A tangent line L is drawn at the point (2, –4) on the parabola $y^2 = 8x$. If the line L is also tangent to the circle $x^2 + y^2 = a$, then 'a' is equal to

Ans. (2)

Sol. Equation of tangent to parabola $y^2 = 8x$ at (2, –4) is $-4y = 4(x + 2)$

$\Rightarrow$ i.e. $x + y + 2 = 0$



OA $\sqrt{a}$

$\left|\frac{0\ \ 0\ \ 2}{\sqrt{2}}\right|\ \ \sqrt{a}$

$\sqrt{2}\ \ \sqrt{a}$

$\Rightarrow a = 2$

3. If S $\frac{7}{5}\ \frac{9}{5^2}\ \frac{13}{5^3}\ \frac{19}{5^4}$ ..... then 160S is equal to

Ans. (305)

Sol. S $\frac{7}{5}\ \frac{9}{5^2}\ \frac{13}{5^3}\ \frac{19}{5^4}$ ..... ……..(1)

$\frac{S}{5}\ \frac{7}{5^2}\ \frac{9}{5^3}\ \frac{13}{5^4}\ \frac{19}{5^5}$ ..... ……..(2)

equation (1) – (2)

$\frac{4S}{5}\ \frac{7}{5}\ \frac{2}{5^2}\ \frac{4}{5^3}\ \frac{6}{5^4}\ \frac{8}{5^5}$ .....

$\frac{4S}{5}\ \frac{7}{5}\ k\ \frac{2}{5^2}\ \frac{4}{5^3}\ \frac{6}{5^4}\ \frac{8}{5^5}$ .....

$\frac{k}{5}\ \frac{2}{5^3}\ \frac{4}{5^4}\ \frac{6}{5^5}\ \frac{8}{5^6}$ .....

$\frac{4k}{5}\ \frac{2}{5^2}\ \frac{2}{5^3}\ \frac{2}{5^4}\ \frac{2}{5^5}$ .....

$\frac{4k}{5}\ \frac{2}{25}\ \frac{1}{1\ \frac{1}{5}}\ \frac{1}{10}\ k\ \frac{1}{8}$ Now $\frac{4S}{5}\ \frac{7}{5}\ \frac{1}{8}\ \ \frac{4S}{5}\ \frac{7}{5}\ \frac{1}{8}$

S $\frac{61}{32}$ 160S 160 $\frac{61}{32}$ 305

4. Suppose the line $\frac{x-2}{5}=\frac{y-2}{2}=\frac{z-2}{\alpha}$ lies on the plane $x+3y-2z+\beta=0$. Then $(\alpha+\beta)$ is equal to

Ans. (7)

Sol. (2,2,–2) lies in plane

$\Rightarrow 2+6+4+\beta=0 \Rightarrow \beta=-12$

$\because$ line is perpendicular to normal

$\Rightarrow \alpha(1)-5(3)+2(-2)=0 \quad \Rightarrow \alpha=19$

So $\alpha+\beta=7$

5. Let B be the centre of the circle $x^2+y^2-2x+4y+1=0$. Let the tangents at two points P and Q on the circle intersect at the point A(3, 1). Then $8\frac{\text{area}(APQ)}{\text{area}(BPQ)}$ is equal to

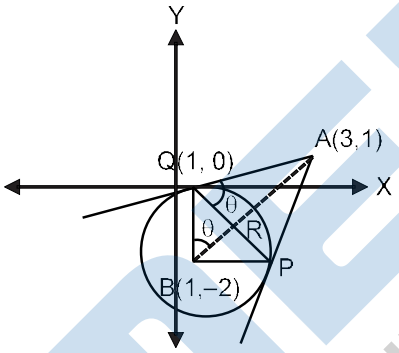
Ans. (18)

Sol. $AP=\sqrt{9-1-6-4-1}$

$AP=3=AQ$

$r=\sqrt{1+4-1}=2$

$\tan\frac{\theta}{2}=\frac{\text{Area}(APQ)}{\text{Area}(BPQ)}=\frac{AR}{RB}=\frac{3\sin\theta}{2\cos\theta}=\frac{9}{4} \quad 8\frac{\text{Area}(APQ)}{\text{Area}(BPQ)}=8$



6. If the coefficient of a^7b^8 in the expansion of $(a+2b+4ab)^{10}$ is $k.2^{16}$, then K is equal to

Ans. (315)

Sol. $a^{10}b^{10}\left(\frac{1}{b}+\frac{2}{a}+4\right)^{10}$

$a^{10}b^{10}\frac{10!\left(\frac{1}{b}\right)^{r_1}\left(\frac{2}{a}\right)^{r_2}4^{10-r_1-r_2}}{r_1!\,r_2!(10-r_1-r_2)!}$

So, $r_1=2$, $r_2=3$

Coefficient of a^7b^8 is $\frac{10!\,2^3\,4^{10-2-3}}{2!\,3!\,5!}=\frac{2^{13}\,10!}{2!\,3!\,5!}=2^{16}\,315$

$k=315$

7. The number of elements in the set $A = \begin{bmatrix} a & b \\ 0 & d \end{bmatrix} : a,b,d \in [-1,0,1]$ and $(I - A)^3 + I = A^3$, where I is 2×2 identity matrix, is

Ans. (8)

Sol. $I - A^3 - 3A + 3A^2 = I - A^3$

$\Rightarrow 3A^2 - 3A = 0 \cdot 3A(A - I) = 0$

$\Rightarrow A^2 = A$

$$\begin{bmatrix} a^2 & ab + bd \\ 0 & d^2 \end{bmatrix} = \begin{bmatrix} a & b \\ 0 & d \end{bmatrix}$$

$\Rightarrow a^2 = a \Rightarrow a = 0,1$

$d^2 = d \Rightarrow d = 0,1$

$b(a + d) = b \Rightarrow b = 0, a + d = 1$

Case I : $b = 0 \Rightarrow (a, d) \equiv (0,1)(0,0), (1,1) \rightarrow 4$ ways

Case II : $a + d = 1 \Rightarrow (1,0)(0,1)$ and $b = \pm 1 \rightarrow 4$ ways

Total = 8 ways

8. The number of 4-digit numbers which are neither multiple of 7 nor multiple of 3 is

Ans. (5143)

Sol. Total 4 digit number = $\underline{9}\ \underline{10}\ \underline{10}\ \underline{10} = 9000$

4 digit numbers divisible 7 (n_7)

1001, 1008, ..., 9996

$9996 = 1001 + (n_7 - 1)7$

$n_7 = 1286$

4 digit number divisible by 3 (n_3)

1002, 1005, ..., 9999

$9999 = 1002 + (n_3 - 1)3$

$n_3 = 3000$

4 digit numbers divisible by 21 (n_{21})

1008, 1031, ..., 9996

$9996 = 1008 + (n_{21} - 1)21$

$n_{21} = 429$

so, 4 digit numbers neither divisible by 7 nor 3

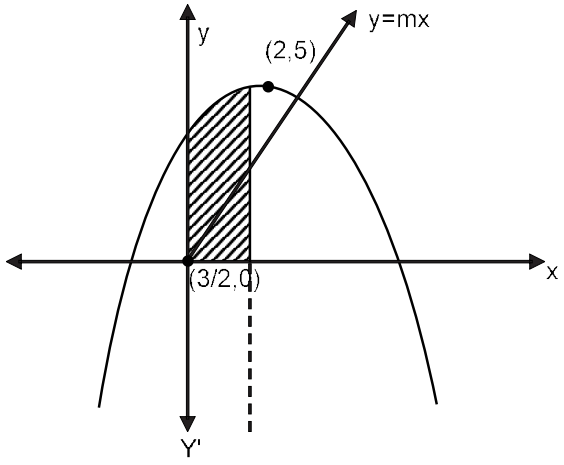
= 9000 – 1286 – 3000 + 429

= 5143

9. If the line y = mx bisects the area enclosed by the line x = 0, y = 0, $x = \frac{3}{2}$ and the curve $y = 1 + 4x - x^2$, then 12m is equal to

Ans. (26)

Sol.



$$\frac{1}{2}\int_0^{3/2}(1+4x-x^2)\,dx = \int_0^{3/2} mx\,dx$$

$$\frac{1}{2}\left[x+2x^2-\frac{x^3}{3}\right]_0^{3/2} = \left[\frac{m}{2}x^2\right]_0^{3/2}$$

$$\frac{3}{2}+\frac{9}{2}-\frac{9}{8} = \frac{9m}{4}$$

$$\frac{39}{8} = \frac{9m}{4}$$

$$m = \frac{39}{18} \quad 12m = \frac{39}{18}\,12$$

$$12m = 26$$

10. Let f(x) be a cubic polynomial with f(1) = –10, f(–1) = 6 and has a local minima at x = 1, and f'(x) has a local minima at x = –1. Then f(3) is equal to

Ans. (22)

Sol. $f(x) = ax^3 + bx^2 + cx + d$

$f'(x) = 3ax^2 + 2bx + c$

$f''(x) = 6ax + 2b$

$f''(-1) = 0 \Rightarrow -6a + 2b = 0 \Rightarrow b = 3a$
$f'(1) = 0 \Rightarrow 3a + 6a + c = 0 \Rightarrow c = -9a$
$f(1) = -10 \Rightarrow -5a + d = -10$……..(i)
$f(-1) = 6 \Rightarrow 11a + d = 6$……..(ii)
(i)–(ii) we get a = 1, d = –5, b = 3, c = –9
Then $f(x) = x^3 + 3x^2 - 9x - 5$

So, f(3) = 27 + 27 – 27 – 5 = 22